

measure in centimetres, and vice versa: you say, “It has thirty centimetres, therefore it has so-and-so many inches.”—Why should one want to translate measurements in terms of centimetres? There may be various reasons. Say cloth is measured by the inch because people generally measure it with their thumbs. But somewhere else it is measured in centimetres, because they have price lists made up in some special way. So we may have reasons for changing the expression of measurement.

You might ask: What are we convinced of when we are convinced of the truth of a logical proposition? How do we become convinced of, say, the law of contradiction?

We first learn a certain technique of using words. Then the most natural continuation for us is to eliminate certain sentences which we don't use—like contradictions. This hangs together with certain other techniques.

Suppose I am a general and I receive reports from reconnaissance parties. One officer comes and says, “There are 30,000 enemy”, and then another comes and says, “There are 40,000 enemy.” Now what happens, or what might happen? I might say, “There are 30,000 soldiers and there are 40,000 soldiers”—and I might go on to behave quite rationally. I might, for instance, act as though there were 30,000, because I knew that one of the soldiers reporting was a liar or always exaggerated. But in fact I should of course say, “Well, one of you must have been wrong”, and I might tell them to go back and look again.

The point is that if I get contradictory reports, then whether you think me rational or irrational depends upon what I do with the reports. If I react by saying, “Well, there are 30,000 and there are 40,000”, you would say, “What on earth do you mean?” You might say, “Surely you can't imagine there being 30,000 *and* 40,000.” But this could be answered in all sorts of ways. I might even draw a picture of it—for instance a blurred picture, or a picture of 30,000 here and of 40,000 there.

“Recognizing the law of contradiction” would come to: acting in a certain way which we call “rational”.

Frege in his preface to the *Grundgesetze der Arithmetik* talks about the fact that logical propositions are not psychological proposi-

tions. That is, we cannot find out the truth of the propositions of logic by means of a psychological investigation—they do not depend on what we think. He asks: What should we say if we found people who made judgments contrary to our logical propositions? What should we say if we found people who did not recognize our logical laws *a priori*, but arrived at them by a lengthy process of induction? Or if we even found people who did not recognize our laws of logic at all and who made logical propositions opposite to ours? He says, “I should say ‘Here we have a new kind of madness’—whereas the psychological logician could only say ‘Here’s a new kind of logic.’ ”<sup>2</sup>

This is queer. We wouldn’t call a man mad who denied the law of contradiction—or would we?

Take this case: people buy firewood by the cubic foot. These people could learn a technique for calculating the price of wood. They stack the wood in parallelepipeds a foot high, measure the length and breadth of the parallelepiped, multiply, and take a shilling for each cubic foot.—This is one way of paying for wood. But people could also pay according to conditions of labour.

But suppose we found people who pile up wood into heaps which are not necessarily a foot high. They measure the length and breadth but not the height, multiply, and say, “The rule is to pay according to the product of length and breadth.” Wouldn’t this be queer? Would you say these people were asking the wrong price? Suppose that in order to show them what a stupid way of calculating the price of wood it is, I take a certain pile which they price at three shillings, and make it longer by making it less high. What if the heap piled differently amounted to £1—and they said, “Well, he’s buying more now, so he must pay more.”—We might call this a kind of logical madness. But there is nothing wrong with giving wood away. So what is wrong with this? We might say, “This is how they do it.”<sup>3</sup>

Another case: Suppose someone wants to find out how many times 3 is contained in this lot of strokes: ||||| |||. Then he may count this way:

2. Page xvi.

3. Cf. *Remarks on the Foundations of Mathematics*, Part I, §§142–152.



“Three, three, three, three—it goes *four* times.”

That seems quite plausible. Suppose people even calculated this way when they wanted to distribute sticks. If nine sticks are to be distributed among three people, they start to distribute four to each. Then one can imagine various things happening. They may be greatly astonished when it doesn't work out. Or they may show no signs of astonishment at all. What should we then say? “We cannot understand them.”

But—and this is an important point—how do we know that a phenomenon which we observe when we are observing human beings is what we ought to call a language? or what we should call calculating?

We most of us talk with the mouth—a few like me with the hands and mouth. And writing is ordinarily done with the hand. And so what we call a language is characterized not merely by its use but by certain other signs too; a criterion of people talking is that they make articulated noises. For instance, if you see me and Watson at the South Pole making noises at each other, everyone would say we were talking, not making music, etc.

Similarly if I see a person with a piece of paper making marks in a certain sort of way, I may say, “He is calculating”, and I expect him to use it in a certain way. Now in the case of the people with the sticks, we say we can't understand these people—because we expect something which we don't find. (If someone came into the room with a bucket on his shoulders, I'd say, “That bucket must hide his head.”)

We can now see why we should call those who have a different logic contradicting ours mad. The madness would be like this: (a) The people would do something which we'd call talking or writing. (b) There would be a close analogy between our talking and theirs, etc. (c) Then we would suddenly see an entire discrepancy between what we do and what they do—in such a way that the whole point of what they are doing seems to be lost, so that we would say, “What the hell's the point of doing this?”

But is there a *point* in everything we do? What is the point of